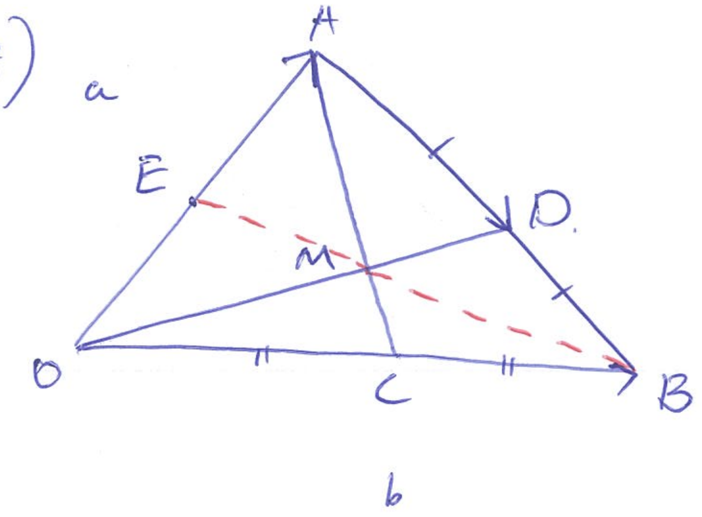
Q8)



$\vec{OC} = \frac{1}{2}b$.

$\vec{AC} = \vec{OC} - \vec{OA} = \frac{1}{2}b - a$, $\vec{AM} = k(\frac{1}{2}b - a)$

$\vec{AB} = \vec{OB} - \vec{OA} = b - a$

$\vec{AD} = \frac{1}{2}\vec{AB} = \frac{1}{2}(b-a)$

$\vec{OD} = \vec{OA} + \vec{AD} = a + \frac{1}{2}b - \frac{1}{2}a = \frac{1}{2}(a+b)$

$\vec{OM} = h\,\vec{OD} = \frac{1}{2}h(a+b)$

$\vec{OA} + \vec{AM} = \vec{OM}$

$a + \frac{1}{2}kb - ka = \frac{1}{2}ha + \frac{1}{2}hb$

$(1-k)a + \frac{1}{2}kb = \frac{1}{2}ha + \frac{1}{2}hb$.

$\begin{cases} 1-k = \frac{1}{2}h \\ \frac{1}{2}k = \frac{1}{2}h \quad \therefore k = h. \end{cases}$ $\quad 1 = \frac{3}{2}h \quad \therefore h = k = \frac{2}{3}$.

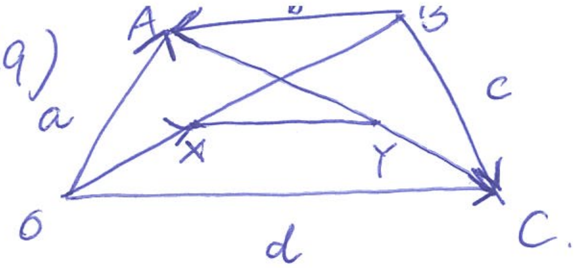
$\therefore \vec{AM} = \frac{2}{3}\vec{AC} \quad \therefore \vec{MC} = \frac{1}{3}\vec{AC} = \frac{1}{3}(\frac{1}{2}b - a) = -\frac{1}{3}a + \frac{1}{6}b$

$\vec{CB} = \vec{OC} = \frac{1}{2}b \qquad \vec{MB} = \vec{MC} + \vec{CB} = -\frac{1}{3}a + \frac{2}{3}b$,

$\therefore \vec{BM} = \frac{1}{3}a - \frac{2}{3}b$

$\vec{BE} = \vec{OE} - \vec{OB} = \frac{1}{2}a - b$,

$\vec{BM} = \frac{2}{3}(\frac{1}{2}a - b) = \frac{2}{3}\vec{BE} \quad \therefore \lambda = \frac{2}{3}$.

Q9) 

$\overrightarrow{OB} = a - b = d - c$ ①

$\overrightarrow{AC} = d - a = c - b$ ②.

$\overrightarrow{OX} = \frac{1}{2}a - \frac{1}{2}b = \frac{1}{2}d - \frac{1}{2}c$

$\overrightarrow{YC} = \frac{1}{2}d - \frac{1}{2}a = \frac{1}{2}c - \frac{1}{2}b$

$$\overrightarrow{OX} + \overrightarrow{XY} + \overrightarrow{YC} = \overrightarrow{OC}$$

$$\left(\frac{1}{2}d - \frac{1}{2}c\right) + \overrightarrow{XY} + \frac{1}{2}c - \frac{1}{2}b = d$$

$$\frac{1}{2}d + \overrightarrow{XY} - \frac{1}{2}b = d$$

$$\overrightarrow{XY} = \frac{1}{2}d + \frac{1}{2}b.$$

$$= \frac{1}{2}(b+d)$$

From ①: $a - b = d - c$. $\quad a + c = b + d$ (*).

$$\overrightarrow{OA} + \overrightarrow{BA} + \overrightarrow{OC} + \overrightarrow{BC}$$

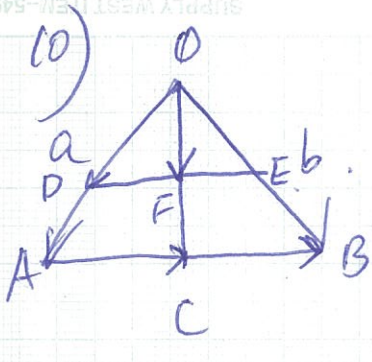
$$= a + (b + d) + c$$

$$= a + (a + c) + c \quad (*)$$

$$= 2(a+c) = 4 \times \frac{1}{2}(b+d) = 4\overrightarrow{XY}$$ □.

P130.

10)



$\vec{OD} = h\,\vec{OA}$ $\quad \vec{OE} = k\,\vec{OB}$ $\quad \vec{OF} = m\,\vec{OC}$

$\quad = h\cdot a \qquad = k\cdot b$

1) $\vec{DF} = ?$

$\vec{AB} = b - a$

$\vec{AC} = \frac{1}{2}b - \frac{1}{2}a = \vec{CB}$

$\vec{OC} = a + \frac{1}{2}b - \frac{1}{2}a = \frac{1}{2}a + \frac{1}{2}b$

$\vec{OF} = m\cdot\frac{1}{2}(a+b)$

$\vec{DF} = \vec{OF} - \vec{OD} = m\cdot\frac{1}{2}a + m\frac{1}{2}b - ha$

$= (\frac{1}{2}m - h)\,a + \frac{1}{2}mb.$

2) $\vec{FE} = \vec{OE} - \vec{OF} = kb - \frac{1}{2}ma - \frac{1}{2}mb$

$= -\frac{1}{2}ma + (k - \frac{1}{2}m)\,b.$

3) $\frac{1}{2}m - h = -\frac{1}{2}m \qquad m = h.$

$\frac{1}{2}m = k - \frac{1}{2}m \qquad m = k.$

$\therefore m = k = h$

4) In $\triangle ODF$ & $\triangle OAC$.

$\frac{OD}{OA} = \frac{OE}{OB} = \frac{OF}{OC} = h = k = m$

$\therefore \triangle ODF \sim \triangle OAC$ (all matching sides are in the same ratio.)

$\therefore \angle ODF = \angle OAC$ (Matching angles are equal)

$\therefore DF \parallel AC$ (corresponding angles are equal)

$\therefore DE \parallel AB$ (D, E, F, & A C B are in the same line.)